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First Quarterly Progress Report
for
COMMAND SYSTEM STUDY FOR THE
OPERATION AND CONTROL OF
UNMANNED SCIENTIFIC SATELLITES
Task I
Unified Tracking/Command/Telemetry
at Lunar Distances
30 June - 30 September 1964

Contract No. NAS 5-9705

Prepared for

Goddard Space Flight Center
Greenbelt, Maryland

Submitted by

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ABSTRACT

This report covers progress during the reporting period on the Command System Study for the Control of Unmanned Scientific Satellites under Task I, Unified Tracking/Command/Telemetry at Lunar Distances. The report reviews the characteristics of the Goddard Range and Range Rate (GRARR) System and the constraints imposed on command signal spectrum and phase deviation by the simultaneous transmission of tracking signals and commands. A comparative analysis is presented of two phase demodulators for command extraction at the spacecraft. Distortion caused by intermodulation between command and tracking signals is treated. Specific conclusions are:

a) Phase demodulation by means of a discriminator-integrator combination is less effective than by means of a synchronous detector. The latter requires carrier extraction at the spacecraft, e.g., a phase-lock loop.

b) Reliable commanding at lunar ranges is possible over the GRARR system with a command phase deviation of 0.5 to 0.72 radian and a data rate of 1200 to 2400 bits per second using a synchronous demodulator in the spacecraft.

c) For simultaneous ranging and commanding the strongest intermodulation products are about 30 db below the major ranging tones. The choice of 10.5 kcps and 16 kcps produces no intermodulation products in the vicinity of any minor ranging tone.

An analysis of double-sideband suppressed carrier modulation is included as a preliminary step for Task III under this contract which is concerned with command system interference.

Author

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I. INTRODUCTION

1.1 General

This report covers work during the first quarter on Task I of Contract NAS 5-9705. A concurrent quarterly report on Task II entitled "Closed-Loop (Feedback) Verification Techniques" has been issued under separate cover. The objectives of Task I are:

> To analyze system requirements for reliable commanding of spacecraft in highly elliptical and lunar anchored orbits. Emphasis will be placed on a "Unified" command/telemetry tracking approach using the existing high-power Goddard Range and Range Rate (GRARR) equipment. This includes use of the down-link for verification and telemetry. The effect of lunar backscatter when commanding in the vicinity of the moon will be studied.

1.2 Summary of Work During Reporting Period

a. The characteristics of the GRARR modulation and ranging signals are reviewed and two possible phase demodulators for updata — a discriminator-integrator and a synchronous detector — are suggested in Sec. 2.1. The effect of the relative position of filters and limiters in the spacecraft is discussed in Sec. 2.2.

b. A comparative analysis of the two types of demodulators is reported in Sec. 2.3, while Sec. 2.4 analyzes the distortion introduced by the synchronous demodulator when commands alone or both commands and ranging signals are present.

c. An analysis of double-sideband suppressed carrier modulation is contained in Sec. 2.5. Although this material more properly belongs under Task III, "Command System Interference", it is included here since no separate report on Task III is to be issued at this time.

II. DISCUSSION

2.1 General Background

The inclusion of an updata signal in the GRARR modulation offers a potentially attractive solution to the updata communication problem with the space vehicle. The attractiveness of such a solution stems from the fact that only a minimum of additional equipment is required in order to fulfill the need for updata communication.

The GRARR signal consists of several ranging tones which are phase modulated onto a carrier. The peak phase deviation of the composite signal is limited by the linear range of the carrier phase modulator to 2.9 radians.

There are several different modes in which the system can operate. One mode of operation uses minor tones at 4.00, 4.008, 4.032, 4.16, and 4.8 kc and major tones at 20 and 100 kc. There is also a possibility of a major tone at 500 kc, but this tone has not presently been employed and there seems to be little reason for using it except in near earth environment.

The minor ranging tones have a modulation index of 0.2 and the major tones have a modulation index of 0.7 radian. Another mode of operation calls for two major ranging tones at 20 and 100 kc and a PN sequence modulating a 4-kc tone. The deviation of the 4-kc tone is 0.45 radian.

A third mode of operation calls for continuous operation of the highest frequency (100 kc) ranging tone with a deviation of 1.06 radians and a sequenced operation of the other tones with a deviation of 0.7 radian. A modification of this mode uses continuous operation of the PN sequence at 4 kc with a deviation of 0.7 radian instead of the sequenced operation of the six lower tones.

None of the planned modes utilize all the allowable deviation. If the first mode is used there is 0.5 radian remaining, with the second mode 1.05 and with the third mode 1.14 radians remaining. It therefore appears possible to include an updata tone with a deviation of 0.5 in the signal transmitted from the ground. In the following we will discuss alternate system configurations, expected range and performance.

The most important link in the proposed updata system is the space vehicle receiver. In Fig. 1 we show the relevant part of the spacecraft receiver. The receiver has an IF amplifier with a bandwidth w_1, wide enough to accommodate the ranging signal with a possible doppler shift and oscillator instabilities. After the first limiter a narrower control channel is taken off. The control channel is used for housekeeping functions within the spacecraft. The phase demodulator used consists of a discriminator followed by an integrator. Since the updata will be narrowband phase-modulated onto the GRARR carrier, an alternate possible demodulator is a synchronous or quadrature detector (SD) as shown in Fig. 2.

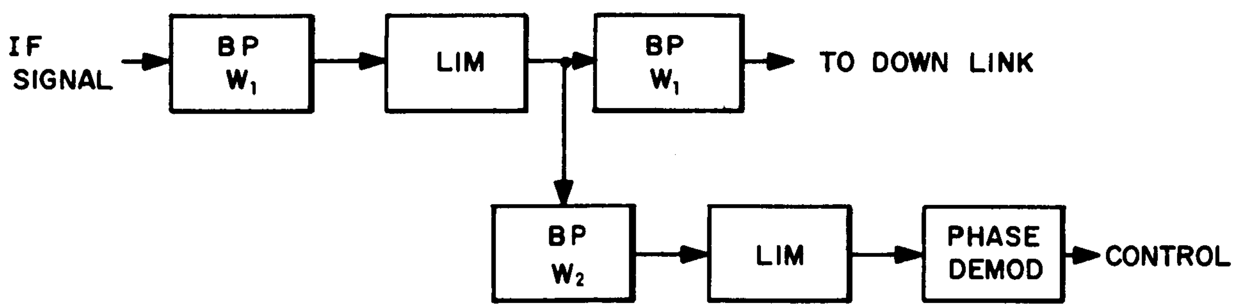

Fig. 1 The relevant part of the spacecraft receiver.

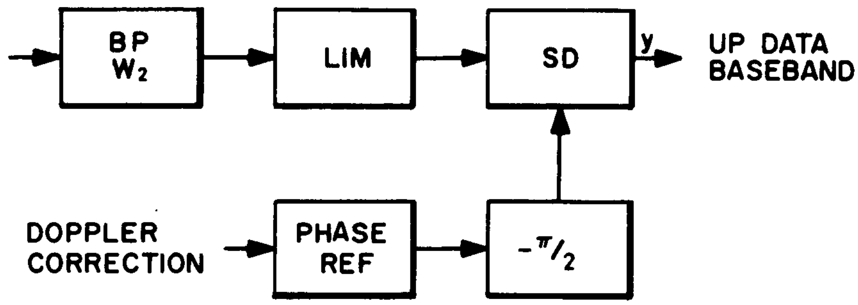

Fig. 2 Quadrature demodulator.

As will be shown in Sec. 2.3.3, for small deviations the quadrature demodulator output is approximately equal to the output from the phase demodulator. There exists one important difference, however. Since the synchronous demodulator is a linear operator it does not suffer from the threshold effect that exists in the discriminator-integrator combination. Since the limiter precedes the demodulator, the overall operation of the demodulator is somewhat degraded. It has been shown by Davenport and others that this degradation is small and only amounts to about 1 db. In our analysis it has been assumed that there is sufficient carrier power for the phase reference to lock on to, and that the subcarriers are switched on in such a sequence as to prevent the reference oscillator from locking on to a subcarrier rather than the actual carrier. These conditions are both fulfilled for the GRARR system.

2.2 Effect of Cascaded Limiters and Filters

Prior to studying the performance of the receiver in Fig. 1 in the presence of noise, it is instructive to inquire as to what the limiter output is if the input is a carrier and an interfering sine wave. We will assume that the interfering signal falls within the bandwidth w_1 but outside the bandwidth w_2. The input signal to the first limiter is therefore

$$e(t) = \mathrm{Re}[A e^{j\omega_c t} + I e^{j\omega_i t}] \tag{1}$$

which can also be expressed as

$$e(t) = I \mathrm{Re}\left\{[1 + \eta e^{j(\omega_c - \omega_i)t}] e^{j\omega_i t}\right\} \tag{2}$$

where $\eta = I/A$, the interference ratio.

The output from the first limiter is e(t) divided by the magnitude of the envelope of e(t).

$$u(t) = Re\left[\frac{1 + \eta e^{j(\omega_c - \omega_i)t}}{\sqrt{|1 + \eta e^{j(\omega_c - \omega_i)t}|^2}} e^{j\omega_i t}\right]. \tag{3}$$

If $\eta < 0.3$, Eq. (3) can be approximated by the use of the two first terms in the expansion for the square root.

$$u(t) \approx Re\left[(1 + \frac{\eta}{2} e^{j(\omega_c - \omega_i)t} - \frac{\eta}{2} e^{-j(\omega_c - \omega_i)t}) e^{j\omega_i t}\right]. \tag{4}$$

Of the three terms shown in the limiter output, only one falls within the passband w_2. The signal input to the second limiter is therefore

$$v'(t) = \frac{\eta}{2} Re[e^{j\omega_c t}] \tag{5}$$

while in the absence of interference it would be

$$v(t) = Re[e^{j\omega_c t}]. \tag{6}$$

If we call S_o the signal input power to the second limiter in the absence of interference, then in the presence of the type of interference described here it is

$$S_i = (\eta/2)^2 S_o. \tag{7}$$

If the gain of the second limiter is sufficient to compensate for variations in signal strength due to the interference ratio $\eta = I/A$, no loss of performance will occur.

If on the other hand the additive noise contributed by the second amplifier limiter becomes comparable to the signal component, this suppression effect can be serious.

It has been shown by Davenport and others that for the range of signal-to-noise ratios under which this system will perform, the effect of cascading limiters will have a very small effect on the performance of the system in additive thermal-type noise. The major consideration is the effect of interference. Since the interference suppresses the thermal noise as well as the signal, the first limiter output signal-to-noise ratio is not affected significantly by this suppression effect. The possible deterioration in performance is due to lack of sufficient gain or excessive noise in the second limiter-amplifier.

Since a certain wide bandwidth, w_1, is required for the ranging signal it is attractive to take advantage of the fact that a narrower bandwidth, w_2, is sufficient for the updata channel. The noise that falls outside of the bandwidth w_2 can therefore be excluded from the updata channel. The noise so excluded is independent of the noise in the band w_2 and is therefore irrelevant as far as the phase demodulator is concerned. This is still a valid assumption even with the first limiter present. Assuming that the noise spectrum is flat over w_1, the gain in signal-to-noise ratio between the ranging channel and the command channel is approximately equal to w_1/w_2.

2.3 A Comparison of Two Updata Demodulators

2.3.1 Introduction

In the following we will analyze the performance of two alternative possible ways of demodulating the updata that has been narrow-band phase modulated on the GRARR system carrier. The two alternatives we will investigate are the phase demodulator consisting of a conventional discriminator followed by an integrator as shown in Fig. 1 and the synchronous quadrature demodulator shown in Fig. 2.

The two data signals representing a binary zero and a binary one are given by

$$\begin{aligned} \text{zero:} \quad \theta_0(t) &= \sin\omega_0 t \\ \text{one:} \quad \theta_1(t) &= \sin\omega_1 t \end{aligned} \tag{8}$$

The resulting receiver IF signal, $i(t)$, is

$$i(t) = Re\left\{ [A e^{jk\theta_i(t)} + \eta(t)] e^{j\omega_c t} \right\} \tag{9}$$

where A is the carrier amplitude, k the modulation index, $\eta(t)$ the complex envelope of the relevant noise, and ω_c the carrier frequency.

Since the noise is of the thermal type and has uniform spectral density over the IF passband, w_2, we obtain the following expression:

$$\overline{Re^2[n(t)]} = \overline{Im^2[n(t)]} = \frac{1}{2}\overline{|n(t)|^2} = N_2 = N_o w_2 \tag{10}$$

where $N_o/2$ is the two-sided spectral density of the noise.

2.3.2 Phase Demodulator

We will start our analysis with a look at the phase demodulator. The demodulator output is given by

$$\theta(t) = \tan^{-1} \frac{A \sin k\theta_i(t) + \text{Im}[n(t)]}{A \cos k\theta_i(t) + \text{Re}[n(t)]} . \tag{11}$$

For IF S/N ratios in excess of 10 db the effect of the noise of Eq. (11) is sufficiently small so that the noise can be treated as a perturbation on the signal, i.e.,

$$\begin{aligned} \theta(t) &\approx k\theta_i(t) + \tan^{-1} \frac{\text{Im}[n(t)]}{A} \\ &\approx k\theta_i(t) + \frac{\text{Im}[n(t)]}{A} \end{aligned} \tag{12}$$

Based on Eq. (12) we can compute the resulting baseband S/N ratio corresponding to a certain IF S/N ratio.

$$(S/N)_{BB} = k^2 \frac{A^2 \overline{\theta_i^2(t)}}{\overline{\text{Im}^2[n(t)]}} = k^2 (S/N)_{IF} \tag{13}$$

Equation (13) shows that the baseband and IF S/N ratios are linearly related with the square of the modulation index as a multiplicative constant. When the IF S/N ratio drops below 10 db, Eq. (12) is no longer a good approximation of the true state of affairs. This is due to the noise steps produced when the magnitude of the noise phasor exceeds A and the phase relationships are such as to produce a step in phase. In the region where $0 < (S/N)_{IF} < 10$ db the following expression is valid.

$$(S/N)_{BB} = m(S/N)_{IF}^{n} \tag{14}$$

where $2 \leq n \leq 3$ and m is a constant depending on k and the realization of the demodulator. 10 db IF S/N ratio is known as the threshold. In the region below threshold Eq. (14) gives evidence of signal suppression by the noise.

To be able to fix the permissible range of baseband S/N ratios we will look more closely at the baseband signal itself and the use to which it is put.

By studying the ranging spectrum shown in Fig. 3 we see that the most suitable region in which to locate the updata carriers is between 8 and 17 kc. The mark and space frequencies should preferably be chosen in such a way that they are not multiples of each other nor multiples of 4 kc or one half of 20 kc. This indicates that sets of frequencies such as 9 and 12 kc or 9 and 15 kc would be suitable choices. However, the choice of data frequencies is also influenced by the possibility of crossmodulation products between data and ranging tones. The limitations imposed by crossmodulation will be examined in Sec. 2.4.

Since it is expected that transmission rates as high as 2400 bits/sec will be required, it seems that the second set, 9 and 15 kc, represents the most judicious choice. Based on a data rate of 2400 bits/sec it seems reasonable to expect that the bandwidth of the mark and space filters will be about 3 kc.

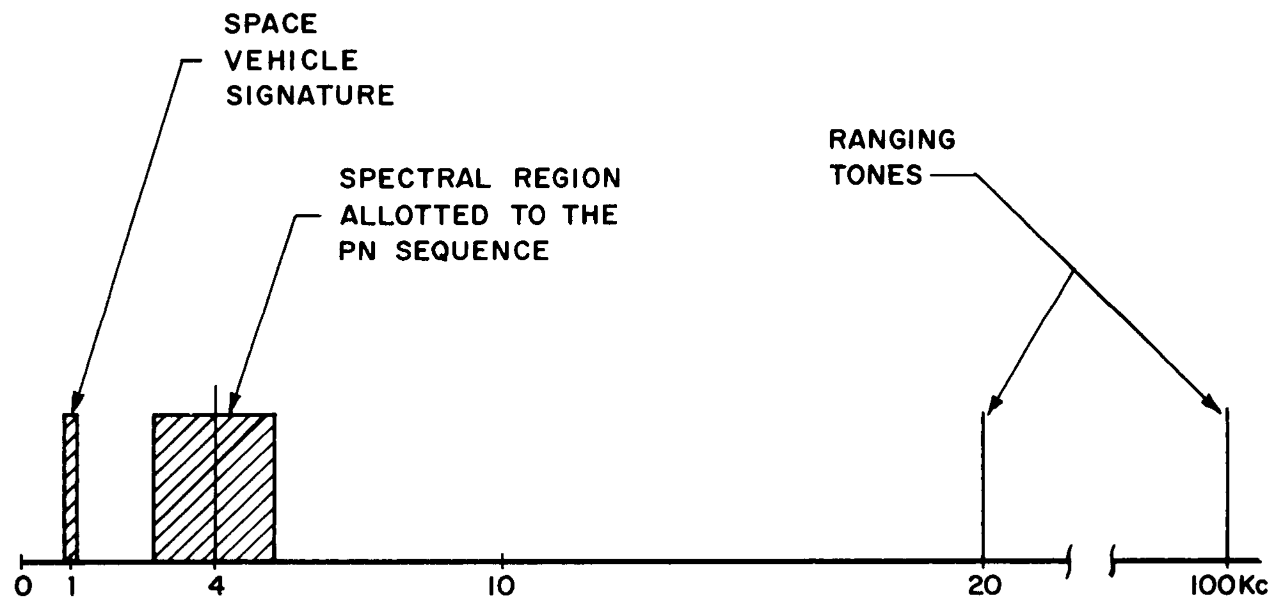

Fig. 3 GRARR baseband spectrum.

The IF S/N ratio of the ranging system at lunar distances is approximately 0 db. The bandwidth of the ranging channel for the S-band system is 400 kc. This bandwidth is required in order to accommodate the ranging information as well as a Doppler shift of ±85 kc. Since the bandwidth of the updata channel, w_2, can be made as narrow as 200 kc, there is a 3-db gain in S/N ratio in the data channel. This still results in a S/N ratio at the phase demodulator of only 3 db. This is 7 db below threshold and will render the channel unusable or, at best, only very marginally usable as an updata link.

2.3.3 Synchronous Demodulator

We will now investigate the performance of the synchronous quadrature demodulator shown in Fig. 2. The signal output from this demodulator is given by

$$y(t) = \frac{B}{2} \sin k\theta_i(t) \tag{15}$$

where B/2 is the demodulator constant. Equation (15) shows that for small k the quadrature demodulator output corresponds to the desired signal. As k increases a certain amount of signal distortion will take place but as long as $k \leq 1$ this distortion will be below a tolerable level. To find the amount of power lost by this nonlinearity of the demodulator, we will consider the case when a zero is transmitted.

$$y_o(t) = \frac{B}{2} \sin(k \sin\omega_o t). \tag{16}$$

For $k \leq 1$ Eq. (16) can be closely approximated by the two first terms in the power series expansion for $\sin\theta$.

$$y_o(t) \approx \frac{B}{2}\left[k \sin\omega_o t - \frac{(k \sin\omega_o t)^3}{3!}\right]$$

$$\approx \frac{B}{2}\left[k \sin\omega_o t - \frac{k^3}{8}\sin\omega_o t + \frac{k^3}{24}\sin 3\omega_o t\right] \tag{17}$$

The power contained in the signal component of the demodulator output is

$$S_o' = \overline{[y_o(t)]^2} = \frac{B^2}{8}(k - \frac{k^3}{8})^2 . \tag{18}$$

The relative received signal power as compared to a conventional phase demodulator is

$$\eta = \frac{S_o'}{S_o} = (1 - \frac{k^2}{8})^2. \tag{19}$$

The relative power loss in db is therefore

$$\eta_{db} = 20 \log_{10}(1 - \frac{k^2}{8}) . \tag{20}$$

To obtain the relevant noise we can take advantage of the fact that the synchronous demodulator is a linear operator. The only relevant part of the noise is therefore only that which falls within the transposed passband of the FSK data demodulator.

The IF noise density equals N/w_1. The total noise in the updata channel passband is therefore $N(w_B/w_1)$ where w_B is the updata channel bandwidth. A gain in signal-to-noise ratio of approximately w_B/w_1 can therefore be expected.

The baseband signal-to-noise ratio for a phase modulated system utilizing synchronous quadrature demodulation is therefore

$$\left(\frac{S}{N}\right)_{BB} \text{db} = 10 \log_{10} \frac{w_1}{w_B} + 20 \log_{10} k \left(1 - \frac{k^2}{8}\right) + \left(\frac{S}{N}\right)_{IF} \text{db} . \qquad (21)$$

Equation (21) is a valid approximation for $k \leq 1$. When we use reasonable parameters for the proposed system, $w_B = 3$ kc, $k = 0.5$, $(S/N)_{IF} = 0$ db, we obtain the following results:

$$(S/N)_{BB} = 21.2 - 6.06 + 0$$

$$(S/N)_{BB} = 15.1 \text{ db} \qquad (22)$$

With this signal-to-noise ratio an actual incoherent FSK system can be expected to have a probability of error of about 10^{-5} at a bit rate of 2400 bits/sec.

This performance can be expected at lunar distances. If a lower probability of error is desired this can be accomplished in two ways. Either one can increase the deviation somewhat (to 0.7 radian) or one can reduce the data rate. Increasing the deviation from 0.5 to 0.72 radian has approximately the same effect on transmission reliability as decreasing the data rate from 2400 to 1200 bits/sec.

As we have seen, the synchronous quadrature demodulator gives a vastly superior performance in this application when compared to the conventional phase demodulator consisting of a discriminator followed by an integrator. Moreover the baseband signal-to-noise ratio is a linear function of IF signal-to-noise ratio even at lunar distances. The performance of this system will therefore degrade gracefully.

2.4 Distortion when Using the GRARR System for Updata

In the following we will study the interaction of updata and ranging signals as well as the effect of demodulator distortion. It has been shown that the maximum range that can be obtained when using the conventional phase demodulator consisting of a discriminator integrator combination is considerably less than that which can be obtained by use of a synchronous quadrature demodulator. We will therefore only consider the distortion in the latter system at this time.

When the baseband signal is of the form

$$\theta_i(t) = \sin \omega_i t \tag{23}$$

the output signal from the synchronous quadrature demodulator is

$$y(t) = \frac{B}{2} \sin (k \sin \omega_i t) \,. \tag{24}$$

Depending on the modulation index, k, there will be a certain amount of harmonic distortion present in the demodulator output.

This distortion is easily visualized when Eq. (24) is expanded in a Fourier series. When this is done we find that the odd terms are Bessel functions while the coefficients of the even terms are zero.

$$y(t) = \sum_{\substack{n=1 \\ (\text{odd})}}^{\infty} J_n(k) \sin n\omega_i t \qquad (25)$$

From Eq. (25) we can readily evaluate the amount of harmonic distortion that will result when a certain deviation is employed. To illustrate this we will examine two cases, k = 0.5 and k = 1.0.

Below is a table showing $J_n(k)$ for k = 0.5 and k = 1.

Table I BESSEL FUNCTIONS

k	$J_1(k)$	$J_3(k)$
0.5	0.2423	0.0026
1.0	0.4401	0.012

From Table I we see that for k = 0.5, y(t) takes on the form

$$y_{k=0.5}(t) = B[0.2423 \sin \omega_i t + 0.0026 \sin 3\omega_i t]. \qquad (26)$$

From Eq. (26) we see that there is a power loss of 0.27 db due to the nonlinear action of the demodulator. The resulting third harmonic distortion is negligible.

As k is increased the relative power loss and the harmonic distortion will also increase. When k = 1 the fundamental signal power loss is 1.1 db. The third harmonic distortion is 16 db below the fundamental. From this distortion analysis and the previous analysis of the performance of the updata link in the presence of thermal-type noise, we conclude that a modulation index, k, in the range 0.5 to 0.75 represents a judicious choice.

At times command and ranging signals may be transmitted simultaneously. When this happens there may be some crossmodulation products introduced into the updata channel as well as some falling too close to the ranging tones.

The combined ranging and updata signal will be of the form

$$v(t) = A \sin\left[\omega_c t + \sum_{n=1}^{N} k_n \sin\omega_n t\right]. \tag{27}$$

In Eq. (27) f_c is the carrier frequency of the transmitted signal while N - 1 is the total number of ranging tones. The Nth tone is the updata subcarrier.

Just as in the case of Eq. (24), Eq. (27) can also be expanded in a Fourier series in order to put the various spectral components of the signal more clearly into view.

When we do this we obtain Eq. (28).

$$v(t) = A \sum_{\ell=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \cdots \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} J_\ell(k_A) J_m(k_B) \cdots J_p(k_D) J_q(k_E)$$

$$\cdot \sin[(\omega_c + \ell\omega_A + m\omega_B \cdots + p\omega_D + q\omega_E)t + \theta] \tag{28}$$

The number of terms in each product in Eq. (28) corresponds to the number of ranging tones in use simultaneously plus one term for the updata subcarrier, f_E. The equation looks rather complicated but the series converges very rapidly so that only relatively few terms need be taken into account.

The peak phase deviation of the ground based transmitter is 2.9 radians. The sidetone combiner is adjusted to provide a modulation index of 0.7 radian for each of the major range tones while the minor ones have a modulation index of 0.2 radian. Since coherent quadrature demodulation is used throughout the system we need only concern ourselves with the odd order harmonics and crossmodulation products. The most serious distortion products are therefore third harmonic distortion and cross-modulation between three subcarriers.

The lowest frequency major ranging tone is 20 kc. The third harmonic of any of the major ranging tones will therefore not fall in the spectral region proposed for the updata subcarriers. If a modulation index of 0.5 is used for the updata subcarrier, the power in the third harmonic of this subcarrier is 63 db below the power in a major ranging tone. The third harmonic of a minor ranging tone is even more insignificant. The only distortion products that are serious enough to warrant some consideration are the products that are formed between one of the minor ranging tones, the updata subcarrier, and the 20-kc ranging tone.

The amplitude of such a crossmodulation term will be down by a factor γ relative to a major ranging tone. With a deviation of 0.5 for the updata tone,

$$\gamma = \frac{J_1(.2)\,J_1(.5)\,J_1(.7)}{J_0(.2)\,J_0(.5)\,J_1(.7)} = 0.026 \;.$$

The power in this type of crossmodulation product, which is the most powerful type, is -31.7 db compared to the power in a major ranging tone. Since the power in a minor ranging tone is - 10.9 db on the same basis, we see that, although such interfering components are not likely to be harmful, they ought not to fall too near the minor ranging tones.

At this juncture we are ready to make the selection of two suitable frequencies for the updata subcarriers. Since the PN sequence will have a spectrum that extends from 0.5 to 7.5 kc, we must choose the updata subcarriers in such a way as not to throw a disturbing crossmodulation product into this region. Since synchronous quadrature demodulators are used throughout the system we only have to worry about crossproducts of odd order. As indicated above, the only crossmodulation term that possesses sufficient amplitude to possibly interfere with a minor ranging tone is the one that falls at the frequency

$$f = \pm f_1 \pm 20 \pm f_i$$

where f_1 is any of the minor range tones and f_i is the updata subcarrier frequency.

A satisfactory set of updata subcarriers both from a spectrum occupancy and a distortion point of view is 10.5 and 16 kc. This set will produce a distortion component at 4.7 kc, but since this frequency is in between the minor ranging tones it is filtered out harmlessly by the ground receiver.

2.5 Use of Double-Sideband Suppressed Carrier Modulation for the Updata Link

A DSBSC system possesses characteristics that make it an interesting alternative to AM or angle modulation for the updata link. In the following we will discuss the basic operation of one possible DSBSC demodulator known as the Costas loop. The performance of the DSBSC system remains unchanged if another type of demodulator possessing equivalent characteristics is used. The DSBSC system is analyzed from the point of view of thermal type noise, interference, and multipath.

2.5.1 The Operation of the DSBSC Demodulator with a Clean Signal

The demodulator that is suitable for demodulating a DSBSC signal is shown in Fig. 4. This demodulator possesses the additional advantage of being able to demodulate a conventional AM signal as well and is in this respect a somewhat universal demodulator. We will start the analysis of the expected performance of a DSBSC system by investigating its operation in the absence of any types of disturbances. Since the demodulator contains three multipliers and only low-pass filters, it is appropriate to use a strictly time domain analysis in our first estimation of system performance. Since the signals are largely sinusoidal in nature, a description based on trigonometric expressions is often convenient.

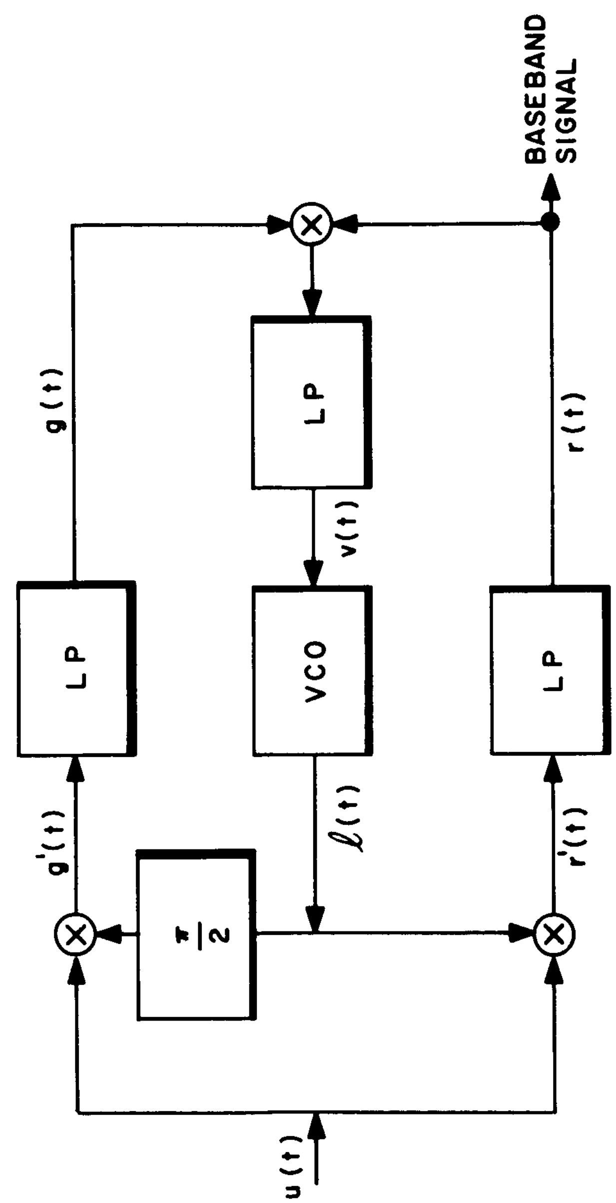

Fig. 4 Synchronous demodulator for a DSBSC system.

An alternate description based on complex envelopes and complex exponentials is often even more convenient since it leads to simplified calculations. Both notations of course express the same phenomena and there is a one-to-one correspondence between the two representations.

The input signal to the demodulator is

$$u(t) = s(t) \cos \omega_c t = \mathrm{Re}[z(t) e^{j\omega_c t}] \tag{29}$$

where

$$s(t) = \mathrm{Re}[z(t)]$$

and

$$\mathrm{Im}[z(t)] = 0 \tag{30}$$

$s(t)$ is the transmitted baseband signal and $z(t)$ is its complex envelope.

The locally generated carrier, $\ell(t)$, is of the form

$$\ell(t) = \cos(\omega_c t + \theta) = \mathrm{Re}[e^{j(\omega_c t + \theta)}] \tag{31}$$

The equations that specify the equilibrium conditions of the feedback loop of the demodulator can now be written down by inspection from Fig. 4.

$$r'(t) = \frac{1}{4} [z(t) e^{j\omega_c t} + z^*(t) e^{-j\omega_c t}][e^{j(\omega_c t + \theta)} + e^{-j(\omega_c t + \theta)}] \tag{32}$$

$$r'(t) = \frac{1}{4} [z(t) e^{j(2\omega_c t + \theta)} + z^*(t) e^{j\theta} + z(t) e^{-j\theta} + z^*(t) e^{-j(2\omega_c t + \theta)}] \tag{33}$$

Since the double frequency terms are not passed by the low-pass filter,

$$r(t) = \frac{1}{4}\left[z(t)\, e^{-j\theta} + z^*(t)\, e^{j\theta} \right] . \tag{34}$$

g(t) can be computed in the same manner, but since the only difference between g(t) and r(t) is that the local carrier has been shifted $\pi/2$ in phase we can use the expression for r(t) if we increase θ by $\pi/2$.

$$g(t) = \frac{1}{4}\left[z(t)\, e^{-j(\theta + \frac{\pi}{2})} + z^*(t)\, e^{j(\theta + \frac{\pi}{2})} \right] \tag{35}$$

or $$g(t) = \frac{j}{4}\left[z^*(t)\, e^{j\theta} - z(t)\, e^{-j\theta} \right] \tag{36}$$

The unfiltered error signal v'(t) is therefore

$$v'(t) = r(t)\, g(t) = \frac{1}{16}\left[z(t)\, e^{-j\theta} + z^*(t)\, e^{j\theta} \right]\left[z(t)\, e^{-j(\theta + \frac{\pi}{2})} + z^*(t)\, e^{j(\theta + \frac{\pi}{2})} \right] . \tag{37}$$

Since $e^{j\frac{\pi}{2}} = j$ and $e^{-j\frac{\pi}{2}} = -j$ this expression can be simplified and becomes

$$v'(t) = \frac{j}{16}\left\{ [z^*(t)]^2\, e^{j2\theta} - z(t)^2\, e^{-j2\theta} \right\} . \tag{38}$$

If this expression is written out in terms of the more familiar trigonometric functions it becomes

$$v'(t) = -\frac{1}{8}\left\{ \left[\mathrm{Re}[z(t)] \right]^2 - \left[\mathrm{Im}[z(t)] \right]^2 \right\} \sin 2\theta + \frac{1}{4} \mathrm{Re}[z(t)]\, \mathrm{Im}[z(t)] \cos 2\theta . \tag{39}$$

The low-pass filter in the feedback loop acts as an integrator which filters out most of the time variable components of the error signal. Since the real and imaginary (in phase and quadrature) components of the signal are usually independent, the VCO control voltage, $v(t)$, becomes

$$v(t) = -\frac{1}{8}\left\{\overline{\left[Re[z(t)]\right]^2} - \overline{\left[Im[z(t)]\right]^2}\right\}\sin 2\theta . \tag{40}$$

From Eq. (40) we see that the control voltage for the VCO has a one-to-one dependence to the phase error θ as it should. We further see that it is possible in the DSBSC system to transmit some information on the quadrature component if necessary.

In a properly operating system θ will of course be close to zero when the system is in equilibrium. In conclusion it is interesting to write down the trigonometric expressions for the in-phase and quadrature signals.

The in-phase signal, $r(t)$, is from Eq. (34)

$$r(t) = \frac{1}{2} Re[z(t)] \cos\theta - \frac{1}{2} Im[z(t)] \sin\theta . \tag{41}$$

The quadrature signal is similarly expressed by use of Eq. (35):

$$g(t) = \frac{1}{2} Im[z(t)] \cos\theta - \frac{1}{2} Re[z(t)] \sin\theta . \tag{42}$$

Since θ will be close to zero for a properly operating system, Eqs. (41) and (42) can therefore, to a good approximation, be written as

$$r(t) = \frac{1}{2} \mathrm{Re}[z(t)] \tag{43}$$

$$g(t) = \frac{1}{2} \mathrm{Im}[z(t)] \tag{44}$$

under proper operating conditions.

2.5.2 DSBSC in White Gaussian Noise

The transmitted signal is of the form

$$s_o(t) \cos\omega_c t = \mathrm{Re}[z(t) e^{j\omega_c t}] \tag{45}$$

where

$$z(t) = A \cos\omega_o t = \frac{A}{2}[e^{j\omega_o t} + e^{-j\omega_o t}] . \tag{46}$$

The average transmitted power in this signal is

$$P_a = \frac{A^2}{4} . \tag{47}$$

The additive Gaussian (thermal type) noise can be expressed as

$$v(t) = \mathrm{Re}[n(t) e^{j\omega_c t}] . \tag{48}$$

$n(t)$ can be expressed most conveniently as a complex Fourier series.

$$n(t) = \sum_{n=-\infty}^{-1} \sqrt{G(f_n)\Delta f}\,(1 - j)\, e^{j[(\omega_n + \omega_c)t - \theta_n]} + \sum_{n=1}^{\infty} \sqrt{G(f_n)\Delta f}\,(1 + j)\, e^{j[(\omega_n - \omega_c)t + \theta_n]} \tag{49}$$

In this expansion $G(f_n)$ is the spectral density of the noise. If the noise is white and has a spectral density $G(f_n) = N_o/2$, $n(t)$ becomes:

$$n(t) = \sum_{n=-\infty}^{-1} \sqrt{N_o \Delta f}\, e^{j[(\omega_n + \omega_c)t - \theta_n - \frac{\pi}{4}]}$$

$$+ \sum_{n=1}^{\infty} \sqrt{N_o \Delta f}\, e^{j[(\omega_n - \omega_c)t + \theta_n + \frac{\pi}{4}]} . \tag{50}$$

The synchronous demodulator operates by multiplying the received signal by $\cos\omega_c t$ and low-pass filtering the output. The signal output from the low-pass filter is therefore

$$s_o'(t) = \frac{A}{2} \cos\omega_o t . \tag{51}$$

The corresponding signal power is

$$\overline{[s_o'(t)]^2} = \frac{A^2}{8} . \tag{52}$$

The output from the synchronous demodulator due to noise is

$$n'(t) = \frac{1}{2} Re[n(t)] . \tag{53}$$

Since the angle θ_n is uniformly distributed over the interval 0 to 2π, the mean square amplitude of the projection of $n(t)$ on the real axis will be down $1/\sqrt{2}$ compared to the mean square amplitude of $n(t)$. The total noise power at the demodulator output is therefore

$$\overline{[n'(t)]^2} = \frac{1}{8} \int_{-w_B}^{w_B} N_o df = \frac{N_o w_B}{4} \tag{54}$$

where w_B is the baseband bandwidth of the system.

The baseband SNR is therefore

$$(S/N)_B = \frac{A^2}{2N_o w_B} \equiv \frac{2P_a}{N_o w_B} . \tag{55}$$

The IF $P_a NR$ is

$$\frac{2P_a}{N_o w_{IF}} . \tag{56}$$

Therefore

$$(S/N)_B \geq 2(S/N)_{IF} . \tag{57}$$

Therefore, the equal sign is valid if w_{IF} is as small as possible, i.e.,

$$w_{IF} = 2w_B .$$

A question that comes up at this point is if the IF SNR or $P_a NR$ has any meaning in the case of a DSBSC system. Since this system does not possess any threshold in the region of interest and the subcarrier SNR is determined by the subcarrier filters, the IF SNR is not a very useful parameter. In systems possessing a threshold behavior, the situation is different since the threshold location is usually a function of IF SNR.

2.5.3 Effect of Multipath on an AM System

We will here consider the effects of multipath disturbances on an AM system. Because of its superior performance, we will assume the receiver to use a synchronous demodulator. This will form an upper bound on the performance we can expect from any AM system. The performance of a receiver using a conventional envelope demodulator will be close to this upper bound for multipath strengths of less than - 6 db. For stronger multipath there will be considerable second harmonic distortion of the signal.

The AM signal is represented by

$$u(t) = A[1 + m(t)] \cos \omega_c t = Re[z(t) e^{j\omega_c t}] . \tag{58}$$

m(t) in Eq. (58) is the baseband modulation and z(t) is the complex envelope of the signal. In this case z(t) is real and is given by

$$z(t) = A[1 + m(t)] . \tag{59}$$

The disturbing reflected signal will be of the same form as u(t) except for an attenuation factor and a time delay. The multipath signal can therefore be described by

$$p(t) = Re[a\, z(t - \tau) e^{j\omega_c (t - \tau)}] . \tag{60}$$

In Eq. (60), a is the voltage reflection coefficient of the multipath producing object and τ is the time delay of the reflected signal relative to the direct one.

We define the phase angle of the reflected signal, θ_τ, as

$$\theta_\tau = - \omega_c \tau + 2\pi n \tag{61}$$

where n is an integer chosen in such a way that

$$|\theta_\tau| < 2\pi . \tag{62}$$

With these definitions we can write the expression for the disturbing reflected signal as

$$p(t) = Re[a z(t - \tau) e^{j\theta_\tau} e^{j\omega_c t}] . \tag{63}$$

The synchronous demodulator depends for its operation on the existence of a locally generated carrier. The phase of this carrier oscillator depends on the phase of the received carrier. Since the receiver input is the linear sum of the direct and reflected signals, the locally generated carrier will be similarly controlled. The received carrier $\ell'(t)$ is

$$\ell'(t) = Re\left\{[\overline{z(t)} + \overline{a z(t - \tau)}\, e^{j\theta_\tau}]\, e^{j\omega_c t}\right\} \tag{64}$$

$$\ell'(t) = A Re[(1 + a e^{j\theta_\tau}) e^{j\omega_c t}] \tag{65}$$

The phase of the local carrier, θ, can be obtained from Eq. (65).

$$\theta = \tan^{-1}\left(\frac{a \sin\theta_\tau}{1 + a\cos\theta_\tau}\right) \tag{66}$$

From Eq. (66) we see that for a strong multipath condition, $0.1 < a < 0.5$, the phase angle of the local oscillator can at most deviate $\pm 20^\circ$ from its proper phase.

The locally generated carrier, $\ell(t)$, is described by

$$\ell(t) = \mathrm{Re}[e^{j(\omega_c t + \theta)}] = \cos(\omega_c t + \theta) . \tag{67}$$

The demodulator output due to the combination of signal and multipath is therefore

$$r(t) = \frac{1}{4}[z(t) + a\,z(t - \tau)\,e^{j\theta_\tau}]\,e^{-j\theta}$$

$$+ \frac{1}{4}[z^*(t) + a\,z^*(t - \tau)\,e^{-j\theta_\tau}]\,e^{j\theta} . \tag{68}$$

$z(t)$ is real so that when this expression is simplified it becomes

$$r(t) = \frac{1}{4}z(t)[e^{j\theta} + e^{-j\theta}] + \frac{1}{4}a\,z(t - \tau)[e^{j(\theta_\tau - \theta)} + e^{-j(\theta_\tau - \theta)}] . \tag{69}$$

The usual trigonometric representation of this expression is easily recognized to be

$$r(t) = \frac{A}{2}[1 + m(t)]\cos\theta + \frac{aA}{2}[1 + m(t - \tau)]\cos(\theta_\tau - \theta) \tag{70}$$

which is the output signal from the synchronous demodulator.

We notice that the main effect of a moderately strong multipath, $a < 0.1$, will be to introduce a delayed replica of the signal into the demodulator output whose strength depends on $\cos(\theta_\tau - \theta)$. From Eq. 66 we see that even a strong reflected signal $a = 0.5$ will have practically no effect on the signal component of the demodulated output since $|\theta| < 20^\circ$.

If the receiver uses an envelope demodulator, the situation is much more complicated. The demodulator output will in this case be

$$r(t) = \sqrt{|z(t) + a\, z(t-\tau)\, e^{j\theta_\tau}|^2} \; . \tag{71}$$

Since z(t) is real, this expression can be written in its trigonometric form as

$$r(t) = A \left\{ [1 + m(t)]^2 + 2a[1 + m(t)][1 + m(t-\tau)] \cos\theta_\tau + a^2 [1 + m(t-\tau)]^2 \right\}^{1/2}$$

If $a \ll 1$ a good approximation for Eq. (72) is

$$r(t) \approx A[1 + m(t)] + aA[1 + m(t-\tau)] \cos\theta_\tau \; . \tag{73}$$

We notice that Eq. (73) is of the same form as Eq. (70) for small a. We therefore conclude that for weak to medium multipath conditions, i.e., $a < 0.1$, the envelope demodulator performs about as well as a synchronous demodulator. For strong multipath conditions there is no simple approximation to Eq. (72) and the resulting demodulator output will therefore be highly dependent on the exact phase relationships.

It does not appear fruitful to extend the investigation further into this region, but in closing we can make the general observation that signal suppression and strong second-harmonic distortion will be present. The synchronous demodulator should therefore be used if strong multipath conditions are anticipated.

2.5.4 DSBSC With Multipath

The DSBSC signal is of the form

$$s(t)\cos\omega_c t = \mathrm{Re}[z(t)\, e^{j\omega_c t}] \, . \tag{74}$$

The disturbing reflected multipath signal can be described by the expression

$$p(t) = \mathrm{Re}[a\, z(t-\tau)\, e^{j\omega_c (t-\tau)}] \tag{75}$$

where a is the voltage reflection coefficient of the multipath producing object and $z(t-\tau)$ is the delayed complex envelope of the transmitted signal.

We will define the phase angle of the multipath signal, θ_τ, as

$$\theta_\tau = -\omega_c \tau + 2\pi n \tag{76}$$

where n is an integer chosen in such a way that

$$|\theta_\tau| < 2\pi \, . \tag{77}$$

By use of the phase angle, θ_τ, we can express the disturbing reflected signal as

$$p(t) = \mathrm{Re}[a\, z(t-\tau)\, e^{j\theta_\tau} e^{j\omega_c t}] \, . \tag{78}$$

The input signal to the demodulator is the linear sum of the direct and reflected signals. The output signal, r(t), is obtained by simple, although somewhat lengthy, calculations. We will assume that the demodulator is properly designed, i. e., that the locally generated carrier, $\ell(t)$, is sufficiently strong so that the signal is handled in a linear manner by the two first multipliers in Fig. 4.

The input signal to the demodulator is

$$u(t) = \mathrm{Re}[z(t)\, e^{j\omega_c t} + a\, z(t-\tau)\, e^{j\theta_\tau} e^{j\omega_c t}] \tag{79}$$

or

$$u(t) = \mathrm{Re}\left\{[z(t) + a\, z(t-\tau)\, e^{j\theta_\tau}]\, e^{j\omega_c t}\right\}. \tag{80}$$

By use of Eq. (38) we can compute the error voltage that will control the VCO.

$$v'(t) = \frac{j}{16}\left\{[z^*(t) + a\, z^*(t-\tau)\, e^{-j\theta_\tau}]^2 e^{j2\theta} - [z(t) + a\, z(t-\tau)\, e^{j\theta_\tau}]^2 e^{-j2\theta}\right\} \tag{81}$$

$$\begin{aligned} v'(t) = \frac{j}{16}\Big\{ & [z^*(t)]^2 e^{j2\theta} + 2a\, z^*(t) z^*(t-\tau)\, e^{-j(\theta_\tau - 2\theta)} \\ & + a^2 [z^*(t-\tau)]^2 e^{-j2(\theta_\tau - \theta)} - [z(t)]^2 e^{-j2\theta} \\ & - 2a\, z(t)\, z(t-\tau)\, e^{j(\theta_\tau - 2\theta)} + a^2 [z(t-\tau)]^2 e^{j2(\theta_\tau - \theta)} \Big\} \end{aligned} \tag{82}$$

$$v'(t) = -\frac{1}{8}\left\{\left[Re[z(t)]\right]^2 - \left[Im[z(t)]\right]^2\right\} \sin 2\theta + \frac{1}{4}R$$

$$+ \frac{1}{4} Re[z(t)]\, Im[z(t)] \cos 2\theta$$

$$- \frac{a}{8}\left\{Re[z(t)]\, Re[z(t-\tau)] - Im[z(t)]\, Im[z(t-\tau)]\right\} \sin(2\theta - \theta_\tau)$$

$$+ \frac{a}{8}\left\{Re[z(t)]\, Im[z(t-\tau)] + Im[z(t)]\, Re[z(t-\tau)]\right\} \cos(2\theta - \theta\tau)$$

$$- \frac{a^2}{8}\left\{\left[Re[z(t-\tau)]\right]^2 - \left[Im[z(t-\tau)]\right]^2\right\} \sin 2(\theta - \theta_\tau)$$

$$+ \frac{1}{4} Re[z(t-\tau)]\, Im[z(t-\tau)] \cos 2(\theta - \theta_\tau) \qquad (83)$$

The meaning of the complex envelope of $z(t-\tau)$ may require some further clarification. Assume that FSK subcarrier modulation is being used and that a tone is being transmitted. In this case

$$Re[z(t)] = Re[s_1 e^{j\omega_1 t}] = s_1 \cos \omega_1 t\,. \qquad (84)$$

In this case the complex envelope of the delayed multipath signal is given by

$$z'(t-\tau) = s_1 e^{j\omega_1 (t-\tau)} \qquad (85)$$

$$= s_1 e^{-j\omega_1 \tau} e^{j\omega_1 t}$$

$$z(t-\tau) = z(t)\, e^{-j\omega_1 \tau} \qquad (86)$$

If, as is usually the case, the quadrature component of the transmitted signal is zero, Eq. (83) can be simplified. The VCO control voltage after filtering becomes

$$
\begin{aligned}
v(t) = &- \frac{1}{8}\overline{\left[Re[z(t)]\right]^2}\sin 2\theta \\
&- \frac{a}{8}\overline{\left[Re[z(t)]\right]^2}\cos\omega_1\tau\sin(2\theta - \theta_\tau) \\
&- \frac{a}{8}\overline{\left[Re[z(t)]\right]^2}\sin\omega_1\tau\cos(2\theta - \theta_\tau) \\
&- \frac{a^2}{8}\left[Re[z(t)]\right]^2(\cos^2\omega_1\tau - \sin^2\omega_1\tau)\sin^2(\theta - \theta_\tau)\ . \qquad (87)
\end{aligned}
$$

Equation (87) is valid for the most usual case where the subcarrier modulation is sinusoidal and the quadrature component of the signal is zero.

When the demodulator is in equilibrium v(t) is close to zero. Equation (87) can therefore be set equal to zero in order to allow one to calculate the phase angle of the local carrier as a result of the multipath disturbance. When this is done one obtains the equilibrium equation for the DSBSC demodulator.

$$
\begin{aligned}
-\sin 2\theta = &\ a\cos\omega_1\tau\sin(2\theta - \theta_\tau) + a\sin\omega_1\tau\cos(2\theta - \theta_\tau) \\
&+ a^2(\cos^2\omega_1\tau - \sin^2\omega_1\tau)\sin 2(\theta - \theta_\tau) \qquad (88)
\end{aligned}
$$

$$
-\sin 2\theta = a\sin(2\theta - \theta_\tau + \omega_1\tau) + \frac{a^2}{2}\sin 2(\theta - \theta_\tau + \omega_1\tau) \qquad (89)
$$

At this point it is convenient to define a baseband delay phase shift, θ_1, as

$$\theta_1 = -\omega_1 \tau + 2\pi m \tag{90}$$

where m is an integer chosen in such a way that

$$|\theta_1| < 2\pi . \tag{91}$$

With this definition the phase equilibrium equation for the DSBSC demodulator in the presence of multipath becomes

$$\sin 2\theta = a \sin(\theta_\tau + \theta_1 - 2\theta) + \frac{a^2}{2} \sin 2(\theta_\tau + \theta_1 - \theta) . \tag{92}$$

This equation is valid for all possible multipath strengths subject only to the previously stated restrictions on the signals used.

Before concluding this section it is necessary to evaluate the effect of the multipath condition on the demodulated baseband signal.

This is easily done by use of Eq. (41) by substituting the actual complex envelope of the received mutilated signal as given by Eq. (80). The resulting expression is

$$\begin{aligned} r(t) &= \frac{1}{2} \mathrm{Re}[z(t) + a z(t-\tau) e^{j\theta_\tau}] \cos\theta \\ &\quad - \frac{1}{2} \mathrm{Im}[z(t) + a z(t-\tau) e^{j\theta_\tau}] \sin\theta . \end{aligned} \tag{93}$$

Considering as before the case of an FSK baseband signal with a complex envelope,

$$z(t) = s_1 [e^{j\omega_1 t} + e^{-j\omega_1 t}] \tag{94}$$

$$r(t) = \frac{1}{2} \mathrm{Re} \left\{ \left[z(t) \right] \left[1 + a e^{j(\theta_\tau + \theta_1)} \right] \right\} \cos\theta$$

$$- \frac{1}{2} \mathrm{Im} \left\{ \left[z(t) \right] \left[1 + a e^{j(\theta_\tau + \theta_1)} \right] \right\} \sin\theta \ , \tag{95}$$

or in the more conventional trigonometric form,

$$r(t) = \frac{1}{2} s_1 \cos\omega_1 t [1 + a\cos(\theta_\tau + \theta_1)] \cos\theta$$

$$- \frac{1}{2} a s_1 \sin\omega_1 t \sin(\theta_\tau + \theta_1) \cos\theta \tag{96}$$

$$r(t) = \frac{1}{2} s_1 [\cos\omega_1 t + a\cos(\omega_1 t + \theta_\tau + \theta_1)] \cos\theta . \tag{97}$$

Equation (97) gives a good picture of the performance of the DSBSC system in the presence of multipath. We see that the effects of moderately strong multipath, i.e., $a < 0.1$, is very slight. Strong multipath, i.e., $0.1 < a < 0.5$, will cause attenuation and phase shift of the received signal but no other distortion effects.

We can therefore conclude that multipath will have approximately the same effect on a DSBSC and an AM system if synchronous demodulation is used. If envelope demodulation is used the performance of the AM system in the presence of strong multipath will be significantly inferior.

2.5.5 DSBSC in a CW Interference Environment

One factor of considerable importance in selecting a modulation system for space communication is its resistance to spurious interference. Here we will investigate the effect of CW-type interference on the DSBSC receiver. Figure 5 shows a block diagram of the synchronous detector.

The input signal is of the form

$$u(t) = \mathrm{Re}\left[s(t)\, e^{j\omega_c t} + I e^{j(\omega_i t + \theta_i)} \right] \tag{98}$$

where I is the amplitude of the interfering signal, and ω_i and θ_i its frequency and phase angle. Equation (98) can be rewritten in the form

$$u(t) = \mathrm{Re}\left\{ \left[s(t) + I e^{j[(\omega_i - \omega_c)t + \theta_i]} \right] e^{j\omega_c t} \right\} . \tag{99}$$

From this equation for the demodulator input we can easily obtain $r(t)$ and $g(t)$ based on the previously developed equations.

$$r(t) = \frac{1}{4} \left\{ [s(t) + I e^{j(\omega_i' t + \theta_i)}] e^{-j\theta} + [s(t) + I e^{-j(\omega_i' t + \theta_i)}] e^{j\theta} \right\} \tag{100}$$

and

$$g(t) = \frac{1}{4} \left\{ [s(t) + I e^{j(\omega_i' + \theta_i)}] e^{-j(\theta + \frac{\pi}{2})} + [s(t) + I e^{-j(\omega_i' t + \theta_i)}] e^{j(\theta + \frac{\pi}{2})} \right\} \tag{101}$$

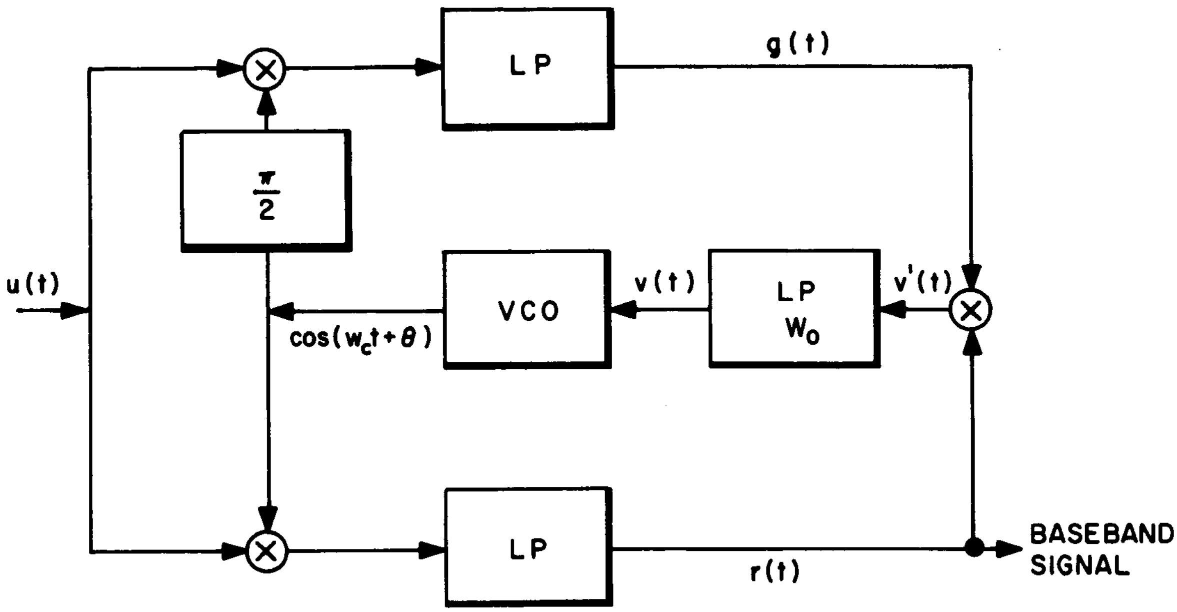

Fig. 5 Synchronous detector for a DSBSC system.

In this development θ is the phase error in the demodulator loop while $\omega_i' = \omega_i - \omega_c$.

The error signal takes on the form

$$v'(t) = g(t)\, r(t)$$

$$v'(t) = \frac{1}{16}\left[z(t)\, e^{-j\theta} + z^*(t)\, e^{j\theta}\right]\left[z(t)\, e^{-j(\theta + \frac{\pi}{2})} + z^*(t)\, e^{j(\theta + \frac{\pi}{2})}\right] \tag{102}$$

Since $e^{j\frac{\pi}{2}} = j$ and $e^{-j\frac{\pi}{2}} = -j$, Eq. (102) can be simplified and becomes

$$v'(t) = \frac{j}{16}\left\{[z^*(t)]^2 e^{j2\theta} - [z(t)]^2 e^{-j2\theta}\right\}. \tag{103}$$

From Eq. (100) we see that z(t) is

$$z(t) = s(t) + I e^{j(\omega_i' t + \theta_i)} \tag{104}$$

$$[z(t)]^2 = [s(t)]^2 + 2 s(t) I e^{j(\omega_i t + \theta_i)} + I^2 e^{j2(\omega_i' t + \theta_i)} \tag{105}$$

$$[z^*(t)]^2 = [s^*(t)]^2 + 2 s^*(t) I e^{-j(\omega_i' t + \theta_i)} + I^2 e^{-j2(\omega_i' t + \theta_i)} \tag{106}$$

Since s(t) is real,

$$s(t) = s^*(t). \tag{107}$$

Equation (103) therefore becomes

$$v'(t) = \frac{j}{16}\left\{[s(t)]^2[e^{j2\theta} - e^{-j2\theta}] + 2 s(t) I[e^{-j(\omega_i' t + \theta_i - 2\theta)} - e^{j(\omega_i t + \theta_i - 2\theta)}] + I^2[e^{-j2(\omega_i' t + \theta_i - \theta)} - e^{+j2(\omega_i' t + \theta_i - \theta)}]\right\}$$

Let the baseband signal be binary FSK. Assume that a zero was transmitted.

$$s(t) = A\cos\omega_o t \tag{108}$$

The resulting error signal can now be written out in trigonometric form as

$$v'(t) = -\frac{1}{8}[(A\cos\omega_o t)^2 \sin 2\theta - 2AI\cos\omega_o t\sin(\omega_i' t + \theta_i - 2\theta)$$

$$- I^2 \sin 2(\omega_i' t + \theta_i - 2\theta)] . \tag{109}$$

After passing through a low-pass filter of bandwidth w_o, only that part of the error signal intended as a control voltage for the VCO is left. It is

$$v'(t) = \frac{1}{16}\left[A^2 \sin 2\theta - 2AI\sin[(\omega_i' - \omega_o)t + \theta_i - 2\theta]\right.$$

$$\left. - I^2 \sin 2(\omega_i' + \theta_i - 2\theta)\right] \tag{110}$$

From Eq. (110) we see that unless the interfering CW signal falls within w_o cps of the carrier frequency or one of the sidebands generated by the modulation process CW interference will not cause any perturbations in the locally generated carrier unless the interference is so strong as to overload the receiver.

III. PROGRAM FOR NEXT REPORTING INTERVAL

a. An investigation will be conducted into the effect of back-scatter from the lunar surface on command reception in the vicinity of the moon.

b. The possibility of a closed-loop command system will be considered employing either the existing telemetry or a separate feedback channel on the PM down-carrier.

c. A Final Report on Task I will be prepared containing recommendations on system configuration and parameters for incorporating updata on the GRARR tracking system. Emphasis will be on the VHF system to be used on the lunar Imp missions.

IV. CONCLUSIONS AND RECOMMENDATIONS

a. Phase demodulation by means of a discriminator-integrator combination is less effective than by means of a synchronous detector. The latter requires carrier extraction at the spacecraft, e.g., a phase-lock loop.

b. Reliable commanding at lunar ranges is possible over the GRARR system with a command phase deviation of 0.5 to 0.72 radian and a data rate of 1200 to 2400 bits per second using a synchronous demodulator in the spacecraft.

c. For simultaneous ranging and commanding the strongest intermodulation products are about 30 db below the major ranging tones. A choice of 10.5 kcps and 16 kcps produces no intermodulation products in the vicinity of any minor ranging tone.